Deterioration Modeling of Structural Members Subjected to Cyclic Loading Using Concentrated Plastic Hinge and Finite-length Plastic-Hinge Models

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Scope

Images adapted from: NIST GCR 10-917-5. “NEHRP Seismic Design Technical Brief No. 4”. 2010
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Concentrated plastic hinge (CPH) formulation

Finite-length plastic-hinge (FLPH) formulation
**Mod IMK Models**

- Simulating the **behavior of structural members** under extreme loading conditions is extremely **complex**, and can only be accomplished by reproducing, at a section level, the **behavior observed experimentally**.

- The use of **empirically calibrated moment-rotation models** that account for **strength and stiffness deterioration** of structural members is paramount in evaluating the performance of steel structures prone to collapse under seismic loading.

- The **Modified Ibarra-Medina-Krawinkler (ModIMK)** models is a complex and general model, which accounts for **six different deterioration mechanisms**, and for that reason, ideal to demonstrate the applicability of the proposed models and for use in collapse structural analysis.
Mod IMK Models

The original Ibarra-Medina-Krawinkler (IMK) model is based on a backbone curve that represents the behavior for monotonic loading and defines the limits for cyclic loading and accounts for four main deterioration modes.

- Basic strength deterioration
- Post-capping strength deterioration
- Unloading stiffness deterioration
- Accelerated reloading stiffness deterioration

Mod IMK Models

- The Modified Ibarra-Medina-Krawinkler (ModIMK), proposed by Lignos and Krawinkler (2008), differs from the original model on some definitions related to the backbone curve and on the simulation of deterioration.

- After the work of Lignos and Krawinkler, that statistically analyzed several hundreds of experimental test results obtained over the last decades, the ModIMK model defines all his parameters, including strength and stiffness deterioration, in terms of element geometry, material properties, and cross-sectional geometry.

\[
\theta_p = 1.65 \cdot \left(\frac{h}{t_w}\right)^{-1.05} \cdot \left(\frac{b_f}{2 \cdot t_f}\right)^{-0.26} \cdot \left(\frac{L_b}{r_y}\right)^{-0.027} \cdot \left(\frac{L}{d}\right)^{0.09} \cdot \left(\frac{d}{c_{unit} \cdot 21''}\right)^{-0.22} \cdot \left(\frac{c_{unit}^2 \cdot F_y}{50}\right)^{-0.15}
\]

- Based on the modes of deterioration defined by the ModIMK model, three different models were implemented in OpenSees.
Mod IMK Models

- Models implemented in OpenSees

- (a) Backbone curve
- (b) Bilin model
- (c) Peak-Oriented mode
- (d) Pinching model

Deterioration mechanisms:
- Basic strength (b,c,d)
- Post-yielding ratio (b,c,d)
- Post-capping strength (b,c,d)
- Unloading Stiffness (b,c,d)
- Reloading stiffness (c,d)
- Pinching(d)
Mod IMK Models

- Modeling the rate of deterioration

The rates of cyclic deterioration are controlled by a characteristic total hysteretic energy dissipation capacity $E_t$ and an energy based rule developed by Rahnama and Krawinkler (1993)

$$\beta_i = \left( \frac{E_i}{E_t - \sum_{j=1}^{c} E_j} \right)^c$$

$E_t = \Lambda \times F_y$

In general, a parameter $X$, which can represent any of the six deterioration modes (e.g., basic strength) and can include a stiffness parameter or a strength parameter, can be updated through:

$$X_i = (1 - \beta_i) \times X_{i-1}$$
**FLPH formulation**

- **What is it?**
  FLPH formulation is an efficient *distributed plasticity formulation* with designated **hinge zones at the member ends**. Cross sections in the **inelastic hinge zones** are characterized through either **nonlinear moment-curvature relationships** or **explicit fiber-section integrations** that enforce the assumption that plane sections remain plane.

- **Integration scheme**
  Scott and Fenves (2006) proposed a **Modified Gauss-Radau** integration scheme in order to **avoid localization issues**.
FLPH formulation

- **Advantages**
  - Avoid localization issues (occurs in distributed plasticity elements)
  - When compared to the concentrated plasticity formulation:
    - Lower modeling effort (less nodes and elements)
    - Lower computational cost
    - Allows for clear separation between member and connection nonlinearity

- **Disadvantages**
  - Not possible to directly use empirically calibrated moment-rotation relationships (such as the ones provided by ModIMK models)
  - Needs a plastic hinge length to be assigned (empirical or based on moment gradient – afternoon)
How to use ModIMK models in FLPH elements?

- Converting a moment-rotation relationship into moment-curvature:

\[
M = K_{M-\chi} \theta = K_{M-\theta} \times L_p
\]

\[
\chi = \frac{\theta}{L_p}
\]

\[
\Lambda_{M-\chi} = \frac{\Lambda_{M-\theta}}{L_p}
\]

- The moment-curvature relationship can then be assigned to define the nonlinear hinge sections.
**FLPH formulation**

- How to use ModIMK models in FLPH elements?
  - However, that is not enough.

\[
K_e = \frac{6EI}{L} L_p
\]

\[
L_p \quad \quad L - 2L_p
\]
**FLPH formulation**

- How to use ModIMK models in FLPH elements?

  - However, that is not enough

Thinking about the elastic region

Original member

\[
\begin{align*}
EI & \\
EI & \\
EI & \\
EI & \\
EI & \\
EI & \\
\end{align*}
\]

FLPH member

\[
\begin{align*}
\frac{6EI}{L} & \\
L_p & \\
EI & \\
EI & \\
EI & \\
\frac{6EI}{L} & \\
L_p & \\
\end{align*}
\]
FLPH formulation

- How to use ModIMK models in FLPH elements?

  ➢ Proposed calibration procedure

FLPH formulation

How to use ModIMK models in FLPH elements?

Proposed calibration procedure

\[ \beta_1 = - \frac{54L_{Pl}L^3 - 6L_{Pl}(60L_{Pl} + 60L_{pJ})L^2 + 6L_{Pl}(96L_{Pl}^2 + 288L_{Pl}L_{pJ} + 96L_{pJ}^2)L - 6L_{Pl}(256L_{pJ}^2L_{pJ} + 256L_{pJ}L^2)}{L(3L - 16L_{pJ})(L^2 - 20LL_{pJ} + 4L_{pJ}L + 64L_{pJ}^2)} \]

\[ \beta_2 = - \frac{3(4L_{pI} - L + 4L_{pJ})(3L^2 - 12LL_{pI} - 12LL_{pJ} + 32L_{pJ}L_{pJ})}{L(3L - 16L_{pI})(3L - 16L_{pJ})} \]

\[ \beta_3 = - \frac{54L_{pJ}L^3 - 6L_{pJ}(60L_{pI} + 60L_{pJ})L^2 + 6L_{pJ}(96L_{pI}^2 + 288L_{pI}L_{pJ} + 96L_{pJ}^2)L - 6L_{pJ}(256L_{pJ}^2L_{pJ} + 256L_{pJ}L^2)}{L(3L - 16L_{pJ})(L^2 - 20LL_{pJ} + 4L_{pJ}L + 64L_{pJ}^2)} \]

If \( L_{pl} = L_{pJ} = L_p \):

\[ \beta_1 = \beta_3 = - \frac{6(3L^2L_p - 24LL_p^2 + 32L_p^3)}{L(L - 8L_p)^2} \]

\[ \beta_2 = 3 \frac{(3L^3 - 48L^2L_p + 224LL_p^2 - 256L_p^3)}{L(3L - 16L_p)^2} \]
FLPH formulation

How to use ModIMK models in FLPH elements?

Proposed calibration procedure – implementation in OpenSees

section Elastic $interiorhingeSection $Es $A_beam [expr $Beta1*$I_beam];
section Elastic $interior $Es $A_beam [expr $Beta2*$I_beam];

set Locations "0 [expr (8.0/3*$Lp_Beam)/$LBeam] [expr (4.0*$Lp_Beam+($LBeam-8*$Lp_Beam)/2)*(1-
1/sqrt(3)))/$LBeam] [expr (4.0*$Lp_Beam+ ($LBeam-8*$Lp_Beam)/2*(1+1/sqrt(3)))/$LBeam] [expr ($LBeam-
8.0/3*$Lp_Beam)/$LBeam] 1.0";
set weights "[expr $Lp_Beam/$LBeam] [expr 3.0*$Lp_Beam/$LBeam] [expr ($(LBeam-8.0*$Lp_Beam)/2)/$LBeam] [expr ((LBeam-8.0*$Lp_Beam)/2) /$LBeam] [expr 3.0*$Lp_Beam/$LBeam] [expr $Lp_Beam/$LBeam]";

set secTags "$hingeSection $interiorhingeSection $interior $interior $interiorhingeSection $hingeSection "$;

set integration "LowOrder 6 $secTags $Locations $weights";

set integration "LowOrder 6 $secTags $Locations $weights";
element forceBeamColumn 8 5 6 $BeamTransfTag $integration ;
**CPH formulation**

- **What is it?**

  - Linear elastic element
  - Rigid-plastic zero-length springs

- **How to overcome the additional flexibility?**
  - Increase the elastic stiffness of the springs
  - Consequently, change the post-yielding ratios
  - Change the stiffness of the element interior

\[ f_b = \frac{L}{6EI} \times \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \]

\[ K_{e,s} = n \frac{6EI}{L} \]

\[ \alpha' = \frac{K_{T,s}}{K_{e,s}} = \frac{\alpha}{1 + n \times (1 - \alpha)} \]

\[ EI_{\text{mod}} = EI \times \frac{n + 1}{n} \]
**CPH formulation**

- **Advantages:**
  - “Directly” assign moment-rotation relationships to the zero-length springs.

- **Disadvantages:**
  - The ideal $n$ value is not trivial, as low values lead to erroneous results and high values result in numerical instability.
  - The use of the parameter $n$, post-yielding ratios does not all Rahnama and Krawinkler (1993) deterioration

\[
X_i = \left( -\beta_0 \right) \times X_{i-1}
\]
CPH formulation

- Problems due to the parameter $n$
  - Take the unloading stiffness update (due to deterioration) as an example

- Elastic stiffness: $K_e$
- Energy dissipated: $E_s = 100 \text{kNm}$
- Deterioration: $\beta_i = \frac{E_i}{E_t - \sum_{j=1}^{i} E_j} = \frac{100}{1000-0} = 0.1$
- Updated unloading stiffness:
  
  $$K_u = (1 - \beta_i) \times K_e = 0.9 \times K_e$$

In the case of a CPH formulation: $K_e = n \times K_e$

Thus:

$$K_u = (1 - \beta_i) \times n \times K_e = 0.9 \times n \times K_e$$

If $n=100$: $K_u = 90 \times K_e$
Problems due to the parameter $n$

Take the unloading stiffness update (due to deterioration) as an example
**CPH formulation**

- Problems due to the parameter $n$

  - Take the unloading stiffness update (due to deterioration) as an example

  - Deterioration parameter $(1 - \beta_i)$ should be seen as a post-yielding ratio thus being computed through:

  \[
  \alpha' = \frac{K_{T,s}}{K_{e,s}} = \frac{\alpha}{1 + n \times (1 - \alpha)}
  \]

  \[
  \alpha' = \frac{K_u}{K_e} = \frac{(1 - \beta_i)}{1 + n \times (1 - (1 - \beta_i))}
  \]
**CPH formulation**

- Problems due to the parameter \( n \)

  Deterioration mechanisms:
  - Basic strength
  - Post-yielding ratio
  - Post-capping strength
  - Unloading Stiffness
  - Reloading stiffness
  - Pinching
**CPH formulation**

- **Proposed implementation**
  - Implemented ModIMK models should be prepared to be used with any formulation

\[
K = (n + 1) \times K_e
\]

\[
\beta_n = \left( \frac{E_i}{E_i - \sum E_i} \right)
\]

\[
\alpha = \frac{\sum E_i}{1 + n \times (1 - \alpha)}
\]

\[
d_{\text{max}, \text{unloading}} = \max(d; d_{\text{max}, \text{of factor}})
\]
**CPH formulation**

- Proposed implementation

  - Unloading stiffness

\[
K_{u,i} = \left( \prod_j^i (1 - \beta_j) \right) \times K_0
\]

In case of CPH formulation:
**CPH formulation**

- Proposed implementation

  ➢ Post-yielding stiffness

```
START

Compute the committed member ratio $\alpha_{i,1}^t$

$$\alpha_{i,1}^t = \frac{\alpha_{i,1}^t (1+n)}{1+n.\alpha_{i,1}^t}$$

Update member ratio $\alpha_i$

$$\alpha_i = \alpha_{i,1} (1-\beta_i)$$

Recompute spring ratio $\alpha_i^t$

$$\alpha_i^t = \frac{\alpha_i}{1+n.(1-\alpha_i)}$$

END```

**CPH formulation**

- **Proposed implementation**
  - **Reloading stiffness**
    
    \[
    d_i = (1 + \beta_i) \times d_{i-1}
    \]

\[d_{spring} = d_{member} - d_{elastic}\]

\[d_{spring} = d_{member} - F(d_{member}) \times F_{member}\]
**CPH formulation**

- Proposed implementation
  - Reloading stiffness

```
START

Compute the committed maximum member displacement \( d_{i-1}^{\text{max, member}} \)

\[
d_{i-1}^{\text{max, member}} = d_{i-1}^{\text{max, spring}} + F^{-1}(d_{i-1}^{\text{max, spring}}) \times L/(6EI)
\]

Update maximum member displacement \( d_i^{\text{max, member}} \)

\[
d_i^{\text{max, member}} = d_{i-1}^{\text{max, member}} \times (1-\beta_i)
\]

Update backbone curve for next iteration (strength, post-yielding ratios)

Recompute the maximum spring displacement \( d_i^{\text{max, spring}} \)

\[
d_i^{\text{max, spring}} = d_i^{\text{max, member}} - F(d_i^{\text{max, member}}) \times L/(6EI)
\]

Update reloading stiffness

END
```
Validation Example

- ModIMK Bilin Model
Validation Example

- ModIMK Peak-Oriented Model
Validation Example

- ModIMK Pinching Model
Conclusions

- Accurate results can be achieved either by using FLPH models or CPH models, since the proposed algorithm is used if the CPH models are employed.

- A significant reduction in model complexity is obtained when FLPH models are employed.

- The implementation procedure for FLPH models is significantly simpler than that required for the CPH models and the use of ad-hoc parameters simulating rigid plastic behavior can be avoided.

Work under development:

- Validate results for different moment gradients.
The Big Picture

Framework for robustness assessment of structures subjected to aftershock hazard events

Step 1
Define performance level

Step 2
Define mainshock hazard

Step 3 – Mainshock Analysis

Step 3.1 – Generate mainshock

Step 3.2 – Development of structural model

Step 3.3 – Damage evaluation due to mainshock only

Step 3.4 – Compute $p_{f1}$

Step 4
Define conditional aftershock hazard

Step 5 – Aftershock Analysis

Step 5.1 – Generate aftershock

Step 5.2 – Damage evaluation due to mainshock-aftershock sequence

Step 5.3 – Compute $p_{f3}$

Step 6
Compute Robustness
The Big Picture

- Building Models: (a) 3-story building LA3, (b) 9-story building LA9, (c) 20-story building LA20
- Buildings designed for the SAC Steel Project for Los Angeles, according to pre-Northridge codes
The Big Picture

Building Models:

- The models assume **rigid diaphragms** at each floor and account for geometric nonlinearities by considering **P–Δ leaning columns**

- **Columns** are modeled considering a **distributed plasticity fiber-section** model an elasto-plastic constitutive law with 3% linear **hardening** assigned to each fiber

- **Beams** are modeled using **finite-length plastic hinge elements** together with a **bilinear** model with **deterioration** based on the empirical model proposed by Lignos & Krawinkler (2011). A plastic hinge length of $L_p = L/6$ (Scott & Ryan 2013) is assigned, which provides for accurate calibration between moment-rotation and moment-curvature relation

- The proposed models were **validated** through nonlinear static (pushover) and nonlinear dynamic time-history analyses against results of past studies (Luco (2002), FEMA355C (2000))
The Big Picture

- A significant correlation between the increase in residual displacements and the reduction in the aftershock leading to failure
Future work

- Compare robustness measures for:
  - Different types of earthquake activity
  - Other Cascading Events:
    - Mainshock – Fire
    - Mainshock – Column Removal – Aftershock
  - Building models with different levels of complexity

- Study different types of structural systems

- Correlation structure between mainshock and aftershock intensities

- Comparative studies between the robustness measures obtained using artificial mainshock-aftershock sequences and real mainshock-aftershock sequences
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